(1,2)*-Generalized Continuous Map in Fuzzy Bitopological Spaces

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Abstract: In this paper, we introduce (1,2)*-Fuzzy generalized continuous maps and (1,2)*-Fuzzy\-
g-continuous maps and study their relations with existing generalized (1,2)*-Fuzzy continuous maps.

Keywords: Fuzzy bitopological space, (1,2)*-fuzzy g- continuous maps, (1,2)*-fuzzy\-g-continuous maps.

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1. Introduction

Levine [9] introduced the concept of Generalized closed sets in topological spaces. Sundaram et al [25] introduced and studied the concept of a class maps namely g- continuous maps which included the continuous maps and a class of gc irresolute maps. Recently Ravi et al [17] Ravi and Thivagar [16] and Duszynski et al [5] introduced (1,2)*-g-closed sets, (1,2)*-sg-closed sets and (1,2)*-f\-g-closed sets in bitopological space respectively. In this paper, we introduce (1,2)*-Fuzzy generalized continuous maps and (1,2)*-Fuzzy -f\-g-continuous maps and study their relations with various generalized (1,2)*-fuzzy continuous maps.

2. Preliminaries

In this section, we list out the definitions and the results which are needed in sequel. Fuzzy sub sets in (X, τ1, τ2) will be denoted by λ, μ, γ and F(X) denotes set of all fuzzy sub sets of (X, τ1, τ2).

Definition 2.1. A fuzzy subset set λ of (X, τ) is called:

(i) Fuzzy semi open (briefly, fs-open) if λ ≤ Cl(Int(λ)) and a fuzzy semi closed (briefly, fs-closed) if Int(Cl(λ)) ≤ λ[1];
(ii) Fuzzy pre open (briefly, fp-open) if A ≤ Int(Cl(λ)) and a fuzzy pre closed (briefly, fp-closed) if Cl(Int(λ)) ≤ λ[4];
(iii) Fuzzy α-open (briefly, fa-open) if λ ≤ IntCl(Int(λ)) and a fuzzy α-closed (briefly, fa-closed) if ClInt(Cl(λ)) ≤ λ[4];
(iv) Fuzzy semi-preopen (briefly, fsp-open) if λ ≤ ClInt(Cl(λ)) and a fuzzy semi-pre closed (briefly, fsp-closed) if IntCl(Int(λ)) ≤ λ[26]. By FSPO (X, τ), we denote the family of all fuzzy semi pre open sets of fts X. The semi closure (resp. α-closure, semi-pre closure ) of a fuzzy set A of (X, τ) is the intersection of all fs-closed (resp. fa-closed, fsp-closed) sets that contain A and is denoted by sCl(λ) (resp. αCl(λ) and spCl(λ)).

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The following lemma is well-known,

**Lemma 2.2.** Let $\lambda$ be a fuzzy set in a fuzzy topological space $(X, \tau)$. Then

(i) $\alpha Cl\lambda = \lambda \lor ClintCl\lambda$.
(ii) $sCl\lambda = \lambda \lor IntCl\lambda$.
(ii) $pCl\lambda \geq \lambda \lor Clint\lambda$.
(iii) $spCl\lambda \geq \lambda \lor IntClint\lambda$.

**Definition 2.3.** A fuzzy set $\lambda$ in a fuzzy topological space $(X, \tau)$ is called:

(i) [2] Fuzzy generalized closed set if $Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is fuzzy open. We briefly denote it as $fg$-closed set.
(ii) [11] Fuzzy semi-generalized closed set if $sCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is fuzzy semi open. We briefly denote it as $fsg$-closed set.
(iii) [8] Fuzzy generalized almost strongly semi-closed set if $\alpha Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is fuzzy $\alpha$-open. We briefly denote it as $f\alpha g$-closed set.
(iv) [23] Fuzzy semi-pre-generalized closed set if $spCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is fuzzy semi-pre-open. We briefly denote it as $fspg$-closed set.
(v) [3] Fuzzy generalized strongly closed set if $\alpha Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is fuzzy open. We briefly denote it as $fg\alpha$-closed set.
(vi) [6] Fuzzy $\bar{g}$-closed set (briefly $fg$-closed set) if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $fsg$-open.
(vii) [7] Fuzzy $\alpha gs$-closed set if $\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is fuzzy semi-open. We briefly denote it as $fas$-closed set.
(viii) [22] Fuzzy generalized semi-closed set if $sCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is fuzzy open. We briefly denote it as $fgs$-closed set.
(ix) [22] Fuzzy generalized semi-pre-closed set if $spCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is fuzzy open. We briefly denote it as $fgsp$-closed set.

**Definition 2.4** ([19]). Let $S$ be a fuzzy subset of a bitopological space $X$. Then $S$ is said to be $\tau_{1,2}$-fuzzy open if $S = \lambda \cup \mu$, where $\lambda \in \tau_1$ and $\mu \in \tau_2$. The complement of $\tau_{1,2}$-fuzzy open set is called $\tau_{1,2}$-fuzzy-closed.

**Definition 2.5.** [19] Let $S$ be a subset of a bitopological space $X$. Then

(i) the $\tau_{1,2}$-closure of $S$, denoted by $\tau_{1,2}$-cl$(S)$, is defined as $\cap\{F : S \subset F$ and $F$ is $\tau_{1,2}$-closed\}.
(ii) the $\tau_{1,2}$-interior of $S$, denoted by $\tau_{1,2}$-int$(S)$, is defined as $\cup\{F : F \subset S$ and $F$ is $\tau_{1,2}$-open\}.

**Definition 2.6.** A fuzzy subset $\lambda$ of a bitopological space $(X, \tau_1, \tau_2)$ is called:

(i) $(1,2)^*$-sg-closed set([16]) if $(1,2)^*$-scl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and $\mu$ is $(1,2)^*$- semi open in $X$. The complement of $(1,2)^*$-sg-closed set is called $(1,2)^*$-sg-open set;
(ii) $(1,2)^*$-gs-closed set([16]) if $(1,2)^*$-scl $(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and $\mu$ is $\tau_{1,2}$-open in $X$. The complement of $(1,2)^*$-gs-closed set is called $(1,2)^*$-gs-open set;
(iii) $(1,2)^*\alpha g$-closed set [18] if $(1,2)^*\alpha cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and $\mu$ is $\tau_{1,2}$-open in $X$. The complement of $(1,2)^*\alpha g$-closed set is called $(1,2)^*\alpha g$-open set;

(iv) $(1,2)^*\hat{g}$-closed set or $f_{\omega}$-closed set [5] if $(1,2)^*\hat{g}cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and $\mu$ is $(1,2)^*$-semi open in $X$. The complement of $(1,2)^*\hat{g}$-closed (resp. $\omega$-closed) set is called $(1,2)^*\hat{g}$-open (resp. $\omega$-open) set;

(v) $(1,2)^*\hat{\omega}$-closed set [13] if $(1,2)^*\hat{\omega}cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and $\mu$ is $(1,2)^*$-$\omega$-open in $X$. The complement of $(1,2)^*\hat{\omega}$-closed set is called $(1,2)^*\hat{\omega}$-open set;

(vi) $(1,2)^*\hat{\alpha}$-closed set [13] if $(1,2)^*\hat{\alpha}cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and $\mu$ is $(1,2)^*$-$\hat{\alpha}$-open in $X$. The complement of $(1,2)^*\hat{\alpha}$-closed set is called $(1,2)^*\hat{\alpha}$-open set;

(vii) $(1,2)^*\hat{g}sp$-closed set [21] if $(1,2)^*\hat{g}spcl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and $\mu$ is $\tau_{1,2}$-open in $X$. The complement of $(1,2)^*\hat{g}sp$-closed set is called $(1,2)^*\hat{g}sp$-open set;

(viii) $(1,2)^*\hat{g}$-closed set [17] if $(1,2)^*\hat{g}cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and $\mu$ is $\tau_{1,2}$-open in $X$. The complement of $(1,2)^*\hat{g}$-closed set is called $(1,2)^*\hat{g}$-open set.

Definition 2.7. A map $f : (X, \tau_{1,2}) \to (Y, \sigma_{1,2})$ is called:

(i) $(1,2)^*g_{\alpha}$-continuous [12] if $f^{-1}(V)$ is a $(1,2)^*g_{\alpha}$-closed set of $X$ for every $\sigma_{1,2}$-closed set $V$ of $Y$.

(ii) $(1,2)^*\psi$-continuous [14] if $f^{-1}(V)$ is a $(1,2)^*\psi$-closed set of $X$ for every $\sigma_{1,2}$-closed set $V$ of $Y$.

(iii) $(1,2)^*\hat{g}$-continuous [5] if $f^{-1}(V)$ is a $(1,2)^*\hat{g}$-closed set of $X$ for every $\sigma_{1,2}$-closed set $V$ of $Y$.

(iv) $(1,2)^*g$-continuous [22] if $f^{-1}(V)$ is a $(1,2)^*g$-closed set of $X$ for every $\sigma_{1,2}$-closed set $V$ of $Y$.

(v) $(1,2)^*\alpha gs$-continuous [20] if $f^{-1}(V)$ is an $(1,2)^*\alpha gs$-closed set of $X$ for every $\sigma_{1,2}$-closed set $V$ of $Y$.

(vi) $(1,2)^*\alpha g$-continuous [10] if $f^{-1}(V)$ is an $(1,2)^*\alpha g$-closed set of $X$ for every $\sigma_{1,2}$-closed set $V$ of $Y$.

(vii) $(1,2)^*\alpha$-continuous [19] if $f^{-1}(V)$ is an $(1,2)^*\alpha$-closed set of $X$ for every $\sigma_{1,2}$-closed set $V$ of $Y$.

(viii) $(1,2)^*\alpha$-continuous [21] if $f^{-1}(V)$ is an $(1,2)^*\alpha$-closed set of $X$ for every $\sigma_{1,2}$-closed set $V$ of $Y$.

3. $(1,2)^*$-Fuzzy generalized closed set

Definition 3.1 [24]. A fuzzy subset $\lambda$ of a fbs $(X, \tau_{1,2})$ called.

(i) $(1,2)^*fsg$-closed set if $(1,2)^*fsgcl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $(1,2)^*$-fuzzy semi open in $X$. The complement of $(1,2)^*fsg$-closed set is called $(1,2)^*fsg$-open set;

(ii) $(1,2)^*fgs$-closed set if $(1,2)^*fsgcl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $\tau_{1,2}$-open in $X$. The complement of $(1,2)^*fgs$-closed set is called $(1,2)^*fgs$-open set;
(iii) $(1,2)^*-fg\alpha$-closed set if $(1,2)^*-\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $\tau_{1,2}$-open in $X$. The complement of $(1,2)^*-fg\alpha$-closed set is called $(1,2)^*-fg\alpha$-open set;

(iv) $(1,2)^*-fj\alpha$-closed set or $f\omega$-closed set if $\tau_{1,2}$-cl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $(1,2)^*-fs\alpha$-open in $X$. The complement of $(1,2)^*-fj\alpha$-closed set is called $(1,2)^*-fj\alpha$-open (resp. $\omega$-open)set;

(v) $(1,2)^*-f\omega$-closed set if $(1,2)^*-sc\lambda(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $(1,2)^*-fg\alpha$-open in $X$. The complement of $(1,2)^*-f\omega$-closed set is called $(1,2)^*-f\omega$-open set;

(vi) $(1,2)^*-f\alpha$-closed set if $(1,2)^*-\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $(1,2)^*-fg\alpha$-open set; (1,2)-fgsp-open set; (1,2)-fgα-closed set if $(1,2)^*-f\alpha$-closed in $X$. The complement of $(1,2)^*-f\alpha$-closed set is called $(1,2)^*-f\alpha$-open set;

(vii) $(1,2)^*-fg\alpha$-closed set if $(1,2)^*-\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $(1,2)^*-fg\alpha$-open in $X$. The complement of $(1,2)^*-fg\alpha$-closed set is called $(1,2)^*-fg\alpha$-open set;

(viii) $(1,2)^*-fg\alpha$-closed set if $(1,2)^*-\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $(1,2)^*-fg\alpha$-open in $X$. The complement of $(1,2)^*-fg\alpha$-closed set is called $(1,2)^*-fg\alpha$-open set.

Definition 3.2 (24). A fuzzy subset $\lambda$ of a fbts $X$ is called $(1,2)^*-fg\alpha$ closed set if $\tau_{1,2}$-cl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $\tau_{1,2}$-open subset in $X$. The complement of $(1,2)^*-fg\alpha$-closed set is called $(1,2)^*-fg\alpha$-open set.

Definition 3.3 (24). A fuzzy subset $\lambda$ of a fbts $X$ is called $(1,2)^*-fj\alpha$-closed set if $\tau_{1,2}$-cl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and $\mu$ is $(1,2)^*-fg\alpha$-open in $X$. The complement of $(1,2)^*-fj\alpha$-closed set is called $(1,2)^*-fj\alpha$-open set.

Proposition 3.4. Every $(1,2)^*-f\alpha$-closed set is $(1,2)^*-f\alpha$-closed.

Proof. If $\lambda$ is a $(1,2)^*-f\alpha$ closed and $\mu$ is any $(1,2)^*-fg\alpha$-open set Containing $\lambda$, we have $\mu \geq \tau_{1,2}$-cl$(\lambda) \geq \alpha cl(\lambda)$. Hence $\lambda$ is $(1,2)^*-f\alpha$-closed in X. □

Proposition 3.5. Every $(1,2)^*-f\alpha$-closed is $(1,2)^*-fg\alpha$-closed

Proof. If $\lambda$ is a $(1,2)^*-f\alpha$-closed subset of $X$ and $\mu$ is any $(1,2)^*-fs\alpha$-open set containing $\lambda$, since every $(1,2)^*-fs\alpha$-open set is $(1,2)^*-fg\alpha$-open, we have $(1,2)^*-\alpha cl(\lambda) \leq \mu$. Hence $\lambda$ is $(1,2)^*-fg\alpha$-closed in X. □

Proposition 3.6. Every $(1,2)^*-fg\alpha$-closed set is $(1,2)^*-f\alpha$-closed.

Proof. If $\lambda$ is $(1,2)^*-f\alpha$-gs closed subset of $X$ and $\mu$ is any $\tau_{1,2}$-open set containing $\lambda$, since every $\tau_{1,2}$-open set is $(1,2)^*-fs\alpha$-open, we have $\alpha cl(\lambda) \leq \mu$. Hence $\lambda$ is $(1,2)^*-f\alpha$ g-closed in X. □

Proposition 3.7. Every $(1,2)^*-f\alpha$-closed set is $(1,2)^*-fj\alpha$-closed.

Proof. If $\lambda$ is a $(1,2)^*-f\alpha$ fuzzy closed subset of $X$ and $\mu$ is any $(1,2)^*-fg\alpha$ open set such that $\mu \geq \lambda = cl(\lambda)$. Hence $\lambda$ is $(1,2)^*-fj\alpha$-closed. □

Proposition 3.8. Every $(1,2)^*-fj\alpha$-closed set is $(1,2)^*-f\omega$-closed.

Proof. If $\lambda$ is a $(1,2)^*-fj\alpha$-closed subset of $X$ and $\mu$ is any $(1,2)^*-fs\alpha$ open set such that $\mu \geq \lambda$, since every $(1,2)^*-fs\alpha$ open set is $(1,2)^*-fg\alpha$ open, we have $\mu \geq sc\lambda(\lambda)$. Hence $\lambda$ is $(1,2)^*-f\omega$-closed. □

Proposition 3.9. Every $(1,2)^*-fj\alpha$-closed set is $(1,2)^*-f\alpha$-closed.
Proof. If $\lambda$ is a $(1,2)^{-}\text{fg}$-closed subset of $X$ and $\mu$ is any $(1,2)^{-}\text{fsg}$-open set such that $\lambda \leq \mu$, then $\mu \geq \text{cl}(\lambda) \geq \alpha \text{cl}(\lambda)$. Hence $\lambda$ is $(1,2)^{-}\text{fg}$α closed.

Proposition 3.10. Every $(1,2)^{-}\text{f}_\omega$-closed set is $(1,2)^{-}\text{fg}$-closed.

Proof. If $\lambda$ is a $(1,2)^{-}\text{f}_\omega$-closed subset of $X$ and $\mu$ is any $(1,2)^{-}\text{f}$-fuzzy open set containing $\lambda$, then $\mu \geq \tau_{1,2^-}\text{cl}(\lambda)$. Hence $\lambda$ is $(1,2)^{-}\text{fg}$-closed in $X$.

Proposition 3.11. Every $(1,2)^{-}\text{f}_\psi$-closed set is $(1,2)^{-}\text{fsg}$ closed.

Proof. If $\lambda$ is a $(1,2)^{-}\text{f}_\psi$-closed subset of $X$ and $\mu$ is any $(1,2)^{-}\text{s}$-open set containing $\lambda$, we have $(1,2)^{-}\text{scl}(\lambda) \leq \mu$. Hence $\lambda$ is $(1,2)^{-}\text{fsg}$-closed.

Proposition 3.12. Every $(1,2)^{-}\text{fg}$-closed set is $(1,2)^{-}\text{fgs}$-closed.

Proof. If $\lambda$ is a $(1,2)^{-}\text{fg}$-closed subset of $X$ and $\mu$ is any $(1,2)^{-}\text{f}_1,2^\alpha$-open set containing $\lambda$, since every $(1,2)^{-}\text{f}_1,2^\alpha$-open set is $(1,2)^{-}\text{fs}$ open, we have $\mu \geq \tau_{1,2^-}\text{cl}(\lambda) \geq (1,2)^{-}\text{scl}(\lambda)$. Hence $\lambda$ is $(1,2)^{-}\text{fg}$-closed in $X$.

Proposition 3.13. Every $(1,2)^{-}\text{f}_\omega$-closed set is $(1,2)^{-}\text{fsg}$-closed.

Proof. If $\lambda$ is a $(1,2)^{-}\text{f}_\omega$-closed subset of $X$ and $\mu$ is any $(1,2)^{-}\text{fs}$ open set containing $\lambda$, we have $\mu \geq \tau_{1,2^-}\text{cl}(\lambda) \geq (1,2)^{-}\text{scl}(\lambda)$. Hence $\lambda$ is $(1,2)^{-}\text{fsg}$-closed in $X$.

Proposition 3.14. Every $(1,2)^{-}\text{fuzzy sg}$-closed set is $(1,2)^{-}\text{fuzzy gs}$-closed.

Proof. If $\lambda$ is $(1,2)^{-}\text{fuzzy sg}$-closed subset of $X$ and $\mu$ is any $(1,2)^{-}\text{f}$-fuzzy semi open set containing $\lambda$, since every $(1,2)^{-}\text{f}$-fuzzy semi open set is $(1,2)^{-}\text{fs}$ open, we have $\text{scl}(\lambda) \leq \mu$. Hence $\lambda$ is $(1,2)^{-}\text{fuzzy gs}$-closed.

Proposition 3.15. Every $(1,2)^{-}\text{fuzzy semi-pre}$-closed set is $(1,2)^{-}\text{fuzzy pre}$-semi-closed.

Proof. If $\lambda$ is a $(1,2)^{-}\text{fuzzy semi pre}$ closed and $\mu$ is any $(1,2)^{-}\text{f}_1,2^\alpha$-open set such that $\lambda \leq \mu$, we have $\text{spCl}(\lambda) = \lambda$, it follows that $\text{spCl}(\lambda) = \lambda \leq \mu$. Hence $\lambda$ is fuzzy pre-semi-closed in $X$.

Proposition 3.16. Every fuzzy pre-semi-closed set is fgsp-closed.

Proof. If $\lambda$ is a $(1,2)^{-}\text{fuzzy pre}$ semi closed and $\mu$ is any $(1,2)^{-}\text{f}_1,2^\alpha$-open set such that $\lambda \leq \mu$, we have $\text{spCl}(\lambda) = \lambda \leq \mu$. Hence $\lambda$ is fgsp-closed in $X$. 

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4. (1,2)*-fuzzy g-continuous Maps and (1,2)*-fuzzy $\tilde{g}$-continuous Maps.

In this section we introduce the following definitions.

Definition 4.1. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called:

(i) $(1,2)^{*}$-fuzzy $g\alpha$-continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy $g\alpha$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(ii) $(1,2)^{*}$-fuzzy $\psi$-continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy $\psi$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(iii) $(1,2)^{*}$-fuzzy $\tilde{g}$-continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy $\tilde{g}$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(iv) $(1,2)^{*}$-fuzzy $\alpha g\alpha s$-continuous if $f^{-1}(V)$ is an $(1,2)^{*}$-fuzzy $\alpha g\alpha s$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(v) $(1,2)^{*}$-fuzzy $\alpha g$-continuous if $f^{-1}(V)$ is an $(1,2)^{*}$-fuzzy $\alpha g$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(vi) $(1,2)^{*}$-fuzzy $gs$-continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy $gs$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(vii) $(1,2)^{*}$-fuzzy $g\alpha s$-continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy $g\alpha s$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(viii) $(1,2)^{*}$-fuzzy $sg$-continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy $sg$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(ix) $(1,2)^{*}$-fuzzy semi-continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy semi-open set of $X$ for every $(1,2)^{*}$-fuzzy open set $V$ of $Y$.

(x) $(1,2)^{*}$-fuzzy $\alpha$-continuous if $f^{-1}(V)$ is an $(1,2)^{*}$-fuzzy $\alpha$-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(xi) $(1,2)^{*}$-fuzzy continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(xii) $(1,2)^{*}$-fuzzy pre semi continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy pre semi-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

(xiii) $(1,2)^{*}$-fuzzy semi pre continuous if $f^{-1}(V)$ is a $(1,2)^{*}$-fuzzy semi pre-closed set of $X$ for every $(1,2)^{*}$-fuzzy closed set $V$ of $Y$.

Definition 4.2. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^{*}$-fuzzy g-continuous if every $f^{-1}(\lambda)$ is $(1,2)^{*}$-fuzzy $g$-closed in $(X, \tau_1, \tau_2)$ for every $(1,2)^{*}$-fuzzy closed set $\lambda$ of $(Y, \sigma_1, \sigma_2)$.

Definition 4.3. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^{*}$-fuzzy g-irresolute if every $f^{-1}(\lambda)$ is $(1,2)^{*}$-fuzzy $g$-closed in $(X, \tau_1, \tau_2)$ for every $(1,2)^{*}$-fuzzy $g$-closed set $\lambda$ of $(Y, \sigma_1, \sigma_2)$.

Definition 4.4. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^{*}$-fuzzy $\tilde{g}$-continuous if every $f^{-1}(\lambda)$ is $(1,2)^{*}$-fuzzy $\tilde{g}$-closed in $(X, \tau_1, \tau_2)$ for every $(1,2)^{*}$-fuzzy $\tilde{g}$-closed set $\lambda$ of $(Y, \sigma_1, \sigma_2)$.

Proposition 4.5. Every $g$-continuous is $(1,2)^{*}$-fuzzy g-continuous but not conversely.

Proof. It is obvious. The converse of the proposition 4.5 is not true as seen from the following example.

Example 4.6. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{0, 1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0.8}{c}\}$, $\tau_2 = \{0, 1, \mu = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.7}{c}\}$. $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^{*}$-fuzzy g-continuous but not $(1,2)^{*}$-fuzzy continuous, since $f^{-1}(\{\gamma = \frac{0.8}{a} + \frac{0.5}{b} + \frac{0.8}{c}\}) = \{\gamma\}$ is not $\tau_1, 2$-closed in $X$.

Proposition 4.7. Every $(1,2)^{*}$-fuzzy $\alpha$-continuous map is $(1,2)^{*}$-fuzzy $\tilde{g}$a-continuous but not conversely.
Proof. The proof follows from Proposition 3.4. The converse of the proposition 4.7 is not true as seen from the following example.

Example 4.8. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4 + 0.2}{\pi}\}$, $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*\text{-fuzzy } \gamma\alpha\text{-continuous but not } (1,2)^*\text{-fuzzy } \alpha\text{-continuous, since} \ f^{-1}\{(\gamma = \frac{0.4 + 0.2}{\pi})\} = \{\gamma\}$ is not $(1,2)^*\text{-fuzzy closed set in } X$.

Proposition 4.9. Every $(1,2)^*\text{-fuzzy } \gamma\alpha\text{-continuous map is } (1,2)^*\text{-fuzzy } \alpha\text{-gs-continuous but not conversely.}$

Proof. The proof follows from Proposition 3.5. The converse of the proposition 4.9 is not true as seen from the following example.

Example 4.10. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4 + 0.2}{\pi}\}$, $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*\text{-fuzzy } \alpha\text{-gs-continuous but not } (1,2)^*\text{-fuzzy } \gamma\alpha\text{-continuous, since} \ f^{-1}\{(\gamma = \frac{0.4 + 0.2}{\pi})\} = \{\gamma\}$ is not $(1,2)^*\text{-fuzzy closed set in } X$.

Proposition 4.11. Every $(1,2)^*\text{-fuzzy } g\text{-continuous map is } (1,2)^*\text{-fuzzy } \alpha\text{-gs-continuous but not conversely.}$

Proof. The proof follows from Proposition 3.12. The converse of the proposition 4.11 is not true as seen from the following example.

Example 4.12. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4 + 0.2}{\pi}\}$, $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*\text{-fuzzy } \gamma\alpha\text{-continuous but not } (1,2)^*\text{-fuzzy } g\text{-continuous, since} \ f^{-1}\{(\gamma = \frac{0.4 + 0.2}{\pi})\} = \{\gamma\}$ is not $(1,2)^*\text{-fuzzy } g\text{-closed in } X$.

Proposition 4.13. Every $(1,2)^*\text{-fuzzy } \alpha\text{-gs-continuous is } (1,2)^*\text{-fuzzy } \alpha\text{-continuous but not conversely.}$

Proof. The proof follows from Proposition 3.6. The converse of the proposition 4.13 is not true as seen from the following example.

Example 4.14. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4 + 0.2}{\pi}\}$, $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*\text{-fuzzy } \gamma\text{-continuous but not } (1,2)^*\text{-fuzzy } \alpha\text{-gs-continuous, since} \ f^{-1}\{(\lambda = \frac{0.4 + 0.2}{\pi})\} = \{\lambda\}$ is not $(1,2)^*\text{-fuzzy } \gamma\alpha\text{-closed in } X$.

Proposition 4.15. Every $(1,2)^*\text{-fuzzy } \alpha\text{-continuous is } (1,2)^*\text{-fuzzy } \gamma\alpha\text{-continuous but not conversely.}$

Proof. The proof follows from Proposition 3.7. The converse of the proposition 4.15 is not true as seen from the following example.

Example 4.16. Let $X = Y = \{a, b, c\}$, $\gamma_1 = \{0, 1, \lambda = \frac{0.7 + 0.3}{\pi}\}$, $\gamma_2 = \{0, 1, \mu = \frac{0.2 + 0.2}{\pi}\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \gamma_1, \gamma_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*\text{-fuzzy } \gamma\text{-continuous but not } (1,2)^*\text{-fuzzy } \gamma\alpha\text{-continuous, since} \ f^{-1}\{(\lambda = \frac{0.2 + 0.2}{\pi})\} = \{\lambda\}$ is not $(1,2)^*\text{-fuzzy } \gamma\alpha\text{-closed set in } X$.

Proposition 4.17. Every $(1,2)^*\text{-fuzzy } \gamma\text{-continuous is } (1,2)^*\text{-fuzzy } \psi\text{-continuous but not conversely.}$

Proof. The proof follows from Proposition 3.8. The converse of the proposition 4.17 is not true as seen from the following example.

Example 4.18. Let $X = Y = \{a, b\}$, $\gamma_1 = \{0, 1, \mu = \frac{0.4 + 0.2}{\pi}\}$, $\gamma_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \gamma_1, \gamma_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*\text{-fuzzy } \psi\text{-continuous but not } (1,2)^*\text{-fuzzy } \gamma\text{-continuous, since} \ f^{-1}\{(\lambda = \frac{0.4 + 0.2}{\pi})\} = \{\lambda\}$ is not $(1,2)^*\text{-fuzzy } \gamma\alpha\text{-closed set in } X$. 
Proposition 4.19. Every $(1,2)^*$-fuzzy $\psi$-continuous is $(1,2)^*$-fuzzy sg continuous but not conversely.

Proof. The proof follows from proposition 3.11. The converse of the proposition 4.19 is not true as seen from the following example.

Example 4.20. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.5}{b}\}$; $\tau_2 \{0, 1\} = F(x)$ and $\tau_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$. Then $f$ is $(1,2)^*$-fuzzy sg-continuous but not $(1,2)^*$-fuzzy $\psi$-continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\lambda\}$ is not $(1,2)^*$-fuzzy $\psi$-closed in $X$.

Proposition 4.21. Every $(1,2)^*$-fuzzy $\omega$-continuous is $(1,2)^*$-fuzzy g-continuous but not conversely.

Proof. The proof follows from proposition 3.10. The converse of the proposition 4.21 is not true as seen from the following example.

Example 4.22. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.5}{a} + \frac{0.5}{b}\}$; $\tau_2 \{0, 1\} = F(x)$ and $\tau_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*$-fuzzy $g$-continuous but not $(1,2)^*$-fuzzy $\omega$-continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\lambda\}$ is not $(1,2)^*$-fuzzy $\omega$-closed in $X$.

Proposition 4.23. Every $(1,2)^*$-fuzzy $\bar{g}$-continuous is $(1,2)^*$-fuzzy $\bar{g}x$-continuous but not conversely.

Proof. The proof follows from proposition 3.9. The converse of the proposition 4.23 is not true as seen from the following example.

Example 4.24. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.5}{b}\}$; $\tau_2 \{0, 1\} = F(x)$ and $\tau_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$. Then $f$ is $(1,2)^*$-fuzzy $\bar{g}x$-continuous but not $(1,2)^*$-fuzzy $\bar{g}$-continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\lambda\}$ is not $(1,2)^*$-fuzzy $\bar{g}$-closed.

Proposition 4.25. Every $(1,2)^*$-fuzzy $g$-continuous is $(1,2)^*$-fuzzy $g\alpha$-continuous but not conversely.

Proof. The proof follows from Proposition 3.13. The converse of the proposition 4.25 is not true as seen from the following example.

Example 4.26. Let $X = Y = \{a, b\}$ and $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.5}{b}\}$; $\tau_2 = F(x)$ and $\tau_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$. Then $f$ is $(1,2)^*$-fuzzy $g\alpha$-continuous but not fuzzy $(1,2)^*$-fuzzy $g$-continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\lambda\}$ is not $(1,2)^*$-fuzzy $g$-closed in $X$.

Proposition 4.27. Every $(1,2)^*$-fuzzy $\omega$-continuous is $(1,2)^*$-fuzzy sg-continuous but not conversely.

Proof. The proof follows from Proposition 3.14. The converse of the proposition 4.27 is not true as seen from the following example.

Example 4.28. Let $X = Y = \{a, b\}$ and $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.5}{b}\}$; $\tau_2 = F(x)$ and $\tau_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$. Then $f$ is $(1,2)^*$-fuzzy sg-continuous but not fuzzy $(1,2)^*$-fuzzy $\omega$-continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\lambda\}$ is not $(1,2)^*$-fuzzy $\omega$-closed $X$.

Proposition 4.29. Every $(1,2)^*$-fuzzy sg-continuous is $(1,2)^*$-fuzzy gs-continuous but not conversely.

Proof. The proof follows from Proposition 3.15. The converse of the proposition 4.29 is not true as seen from the following example.
Example 4.30. Let $X=Y=\{a, b\}$ and $\tau_1 = \{0, 1, \mu = \frac{0.3}{x} + \frac{0.6}{y}\}, \tau_2 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*$-fuzzy $gs$-continuous but not fuzzy $(1,2)^*$-fuzzy $sg$-continuous, since $f^{-1}\{\lambda = \frac{0.3}{x} + \frac{0.4}{y}\} = \{\lambda\}$ is not $(1,2)^*$-fuzzy $sg$-closed $X$.

Proposition 4.31. Every $(1,2)^*$-fuzzy $\omega$-continuous is $(1,2)^*$-fuzzy $ags$-continuous but not conversely.

Proof. The proof follows from Proposition 3.16. The converse of the proposition 4.31 is not true as seen from the following example.

Example 4.32. Let $X=Y = \{a, b\}$ and $\tau_1 = \{0, 1, \mu = \frac{0.3}{x} + \frac{0.7}{y}\}, \tau_2 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*$-fuzzy $ags$-continuous but not fuzzy $(1,2)^*$-fuzzy $\omega$-continuous, since $f^{-1}\{\lambda = \frac{0.3}{x} + \frac{0.4}{y}\} = \{\lambda\}$ is not $(1,2)^*$-fuzzy $\omega$-closed $X$.

Proposition 4.33. Every $(1,2)^*$-fuzzy semi pre continuous is $(1,2)^*$-fuzzy pre semi continuous but not conversely.

Proof. The proof follows from Proposition 3.17.

The converse of the proposition 4.33 is not true as seen from the following example.

Example 4.34. Let $X=Y = \{a, b, c\}$, $\tau_1 = \{0, 1, \lambda = \frac{0.7}{x} + \frac{0.3}{y} + \frac{1}{z}\}, \tau_2 = \{0, 1, \mu = \frac{0.7}{x} + \frac{0.3}{y} + \frac{0}{z}\}$. $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*$-fuzzy semi pre continuous but not $(1,2)^*$-fuzzy semi pre continuous, since $f^{-1}\{\gamma = \frac{1}{x} + \frac{0.3}{y} + \frac{0}{z}\} = \{\gamma\}$ is not fuzzy semi pre closed in $X$.

Proposition 4.35. Every $(1,2)^*$-fuzzy pre semi continuous is fgsp continuous but not conversely.

Proof. The proof follows from Proposition 3.18.

The converse of the proposition 4.35 is not true as seen from the following example.

Example 4.36. Let $X=Y = \{a, b, c\}$, $\tau_1 = \{0, 1, \lambda = \frac{1}{x} + \frac{0}{y} + \frac{0}{z}\}, \tau_2 = \{0, 1\}$. $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then $f$ is $(1,2)^*$-fgsp continuous but not $(1,2)^*$-fuzzy pre semi continuous, since $f^{-1}\{\gamma = \frac{1}{x} + \frac{1}{y} + \frac{2}{z}\} = \{\gamma\}$ is not fuzzy semi pre closed in $X$.

Remark 4.37. From the above Examples 4.6, 4.8, 4.10, 4.12, 4.14, 4.16, 4.18, 4.20, 4.22, 4.24, 4.26, 4.28, 4.30, 4.32, 4.34, 4.36 and propositions 4.5, 4.7, 4.9, 4.11, 4.13, 4.15, 4.17, 4.19, 4.21, 4.23, 4.25, 4.27, 4.29, 4.31, 4.33, 4.35 we obtain the following diagram, where $A \rightarrow B$ (resp. $A \Leftrightarrow B$) represents $A$ implies $B$ but not conversely (resp. $A$ and $B$ are independent of each other).

```
1 2 3 4
5 6 7 8
9 10 11 12
```

Where
(1) (1,2)*-fω-continuous  
(2) (1,2)*- fijg-continuous  
(3) (1,2)*- fgs-continuous  
(4) (1,2)*- fg-continuous  
(5) (1,2)*- fuzzy -continuous  
(6) (1,2)*-fg-continuous

Definition 4.38. For every set A \subseteq X, we define the (1,2)*-g-closure of A to be the intersection of all (1,2)*-fg-closed sets containing A. In symbols, (1,2)*-g-cl(A) = \cap \{ F : A \subseteq F \in (1,2)*-FGC(X) \}.

Proposition 4.39. A map f : (X, τ1, τ2) → (Y, σ1, σ2) is (1,2)*-fg-continuous if and only if f−1(U) is (1,2)*-fg-open in X for every σ1,2-open set U in Y.

Proof. Let f : (X, τ1, τ2) → (Y, σ1, σ2) be (1,2)*-fg-continuous and U be an σ1,2-open set in Y. Then U is σ1,2-closed in Y and since f is (1,2)*-fg-continuous, f−1(U) is (1,2)*-fg-closed in X. But f−1(U) is (1,2)*-fg-open in X. Conversely, assume that f−1(U) is (1,2)*-fg-open in X for every σ1,2-open set U in Y. Let F be a σ1,2-closed set in Y. Then F is σ1,2-open in Y and by assumption, f−1(F) is (1,2)*-fg-open in X. Since f−1(F) is (1,2)*-fg-closed in X and so f is (1,2)*-fg-continuous.

Remark 4.40. The composition of two (1,2)*-fg-continuous maps need not be (1,2)*-fg-continuous and this is shown by the following example.

Example 4.41. Let X = Y = Z=\{ a, b \}, τ1 = 0,1, \lambda = \frac{0.4}{1}, \mu = \frac{0.3}{1}, \tau2 = 0,1. Then τ12-open sets are 0,1, λ = \frac{0.4}{1}, \mu = \frac{0.3}{1} and τ12-closed sets are 0,1, X = \frac{0.6}{1}, \frac{0.4}{1}. Then τ12-open sets are 0,1, μ = \frac{0.3}{1}, \mu = \frac{0.7}{1} and τ12-closed sets are 0,1, \phi = \frac{0.7}{1}, \frac{0.4}{1}.

Let σ1 = 0,1, \eta = \frac{0.4}{1}, \frac{0.5}{1}. Then σ12-open sets are 0,1, \eta = \frac{0.4}{1}, \frac{0.5}{1} and σ12-closed sets are 0,1, \phi = \frac{0.7}{1}, \frac{0.4}{1}. Let f = \{ X, τ1, τ2 \} → (Y, \sigma1, \sigma2) and g : (Y, \sigma1, \sigma2) → (Z, \eta1, \eta2) be the identity maps. Then f and g are (1,2)*-fg-continuous but their composition g o f : (X, τ1, τ2) → (Z, \eta1, \eta2) is not (1,2)*-fg-continuous, because V = \frac{0.4}{1} + \frac{0.6}{1} is \eta1,2-closed in Z but (g o f)−1(V) = f−1(g−1(V)) = f−1(V) = V = \frac{0.4}{1} + \frac{0.6}{1} which is not (1,2)*-fg-closed in X.

Proposition 4.42. If f : (X, τ1, τ2) → (Y, \sigma1, \sigma2) is (1,2)*-fg-continuous and g : (Y, \sigma1, \sigma2) → (Z, \eta1, \eta2) is (1,2)*-continuous, then their composition g o f : (X, τ1, τ2) → (Z, \eta1, \eta2) is (1,2)*-fg-continuous.

Proof. Let F be any \eta1,2-closed set in (Z, \eta1, \eta2). Since g : (Y, \sigma1, \sigma2) → (Z, \eta1, \eta2) is (1,2)*-continuous, g−1(F) is \sigma1,2-closed in (Y, \sigma1, \sigma2). Since f : (X, τ1, τ2) → (Y, \sigma1, \sigma2) is (1,2)*-fg-continuous, f−1(g−1(F)) is (1,2)*-fg-closed in X and so g o f is (1,2)*-fg-continuous.

Proposition 4.43. If A is (1,2)*-fg-closed in X and if f : (X, τ1, τ2) → (Y, \sigma1, \sigma2) is (1,2)*-fg-irresolute and (1,2)*-fuzzy closed, then f(A) is (1,2)*-fg-closed in Y.
Proof. Let U be any \((1,2)*\)-fsg-open in Y such that \(f(A) \subseteq U\). Then \(A \subseteq f^{-1}(U)\) and by hypothesis, \(\tau_{1,2}\text{-cl}(A) \subseteq f^{-1}(U)\). Thus \(f(\tau_{1,2}\text{-cl}(A)) \subseteq U\) and \(f(\tau_{1,2}\text{-cl}(A))\) is a \(\sigma_{1,2}\)-closed set. Now, \(\sigma_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) = f(\tau_{1,2}\text{-cl}(A)) \subseteq U\). That is \(\sigma_{1,2}\text{-cl}(f(A)) \subseteq U\) and so \(f(A)\) is \((1,2)*\)-fg-closed.

**Definition 4.44.** Let \(x\) be a point of \(X\) and \(A\) be a subset of a fuzzy bitopological space \(X\). Then \(x\) is called an \((1,2)*\)-fg-neighbourhood of \(x\) (briefly, \((1,2)*\)-fg-nbhd of \(x\)) in \(X\) if there exists an \((1,2)*\)-fg-open set \(U\) of \(X\) such that \(x \in U\). Theorem 4.46. Suppose the collection of all \((1,2)*\)-fg-open sets of \(X\) is closed under arbitrary unions. Let \(f : (X, \sigma_{1,2}) \to (Y, \sigma_{1,2})\) be a map from a fuzzy bitopological space \((X, \tau_{1,2})\) into a fuzzy bitopological space \((Y, \tau_{1,2})\). Then the following statements are equivalent.

(i) The function \(f\) is \((1,2)*\)-fg-continuous.

(ii) The inverse of each \(\sigma_{1,2}\)-open set is \((1,2)*\)-fg-open.

(iii) For each point \(x\) in \(X\) and each \(\sigma_{1,2}\)-open set \(V\) in \(Y\) with \(f(x) \in V\), there is an \((1,2)*\)-fg-open set \(U\) in \(X\) such that \(x \in U\), \(f(U) \subseteq V\).

(iv) The inverse of each \(\sigma_{1,2}\)-closed set is \((1,2)*\)-fg-closed.

(v) For each \(x\) in \(X\), the inverse of every neighborhood of \(f(x)\) is an \((1,2)*\)-fg-nbhd of \(x\).

(vi) For each \(x\) in \(X\) and each neighborhood \(N\) of \(f(x)\), there is an \((1,2)*\)-fg-nbhd \(G\) of \(x\) such that \(f(G) \subseteq N\).

(vii) For each subset \(A\) of \(X\), \(f(\sigma_{1,2}\text{-cl}(A)) \subseteq \sigma_{1,2}\text{-cl}(f(A))\).

(viii) For each subset \(B\) of \(Y\), \((1,2)*\)-fg-cl\((f^{-1}(B)) \subseteq f^{-1}(\sigma_{1,2}\text{-cl}(B))\).

**Proof.**

(i) \(\Leftrightarrow\) (iii). Suppose that (iii) holds and let \(V\) be an \(\sigma_{1,2}\)-open set in \(Y\) and let \(x \in f^{-1}(V)\). Then \(f(x) \in V\) and thus there exists an \((1,2)*\)-fg-open set \(U_x\) such that \(x \in U_x\) and \(f(U_x) \subseteq V\). Now, \(x \in U_x \subseteq f^{-1}(V)\) and \(f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x\). Then \(f^{-1}(V)\) is \((1,2)*\)-fg-open in \(X\) and therefore \(f\) is \((1,2)*\)-fg-continuous.

Conversely, suppose that (i) holds and let \(f(x) \in V\) where \(V\) is \(\sigma_{1,2}\)-open in \(Y\). Then \(f^{-1}(V) \in (1,2)*\text{-GO}(X)\), since \(f\) is \((1,2)*\)-fg-continuous. Let \(U = f^{-1}(V)\). Then \(x \in U\) and \(f(U) \subseteq V\).\]
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